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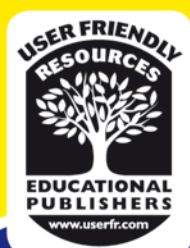
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Tel: 64 3 3772886; Fax: 64 3 3772086; email: [info@userfr.com](mailto:info@userfr.com)  
Thank you.

# **creative Maths Problems**

**Awakening Confidence  
and Enthusiasm  
Book A**

**Numbers, Measurement & Geometry**

**Nicolla Hansen**





## TITLE

Book Name: Creative Maths Problems  
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# Introduction

**M**y motivation for developing this resource stems from a personal ambition to help students experience the joy and satisfaction that can come from mathematical problem solving. Through observation and experimentation in the maths classroom, I believe I have gained some insight into strategies that work and strategies that do not work. Above all it seems to me that the recipe for success is providing students with work that they perceive as achievable and fun. I hope that the problems in this book will be a source of motivation for students, encouraging them to problem solve with confidence and enthusiasm.

In developing this resource, I was conscious of the fact that mathematic education places a huge emphasis on students demonstrating sound mathematical process in their approach to problems. Teachers are being encouraged to teach students to think logically, apply reason and communicate effectively. Mathematical problem solving has the potential to teach these skills, but in my opinion, educators need to proceed with some caution. Too often, I have seen students become frustrated and disillusioned with maths problems because they lack the skills and the confidence to solve them. It is my hope that this book will assist students in acquiring the skills necessary to confidently investigate any mathematical problem they are presented with. In particular the suggestions for teachers, which accompany each problem, are designed to help teachers teach students to approach the problems confidently, strategise appropriately and gain a deeper understanding of the mathematical process involved.

There are also opportunities for students to engage in cooperative learning, pose new problems for investigation or enter into classroom discussion and debate. These are generic skills that today's society is demanding from young people leaving the education system.

The problems in this book have been organised into six areas within mathematics:

- Number
- Measurement
- Geometry
- Algebra
- Statistics
- Logic





The “star” key is intended as a quick way for teachers to gauge the appropriateness of a particular problem. It may also be used as a motivational tool for students:

Key to the difficulty of problems:

- ☆ Suitable for all students
- ☆☆ Suitable for students who enjoy independent learning
- ☆☆☆ Suitable for students who are confident mathematicians.

The problems have the potential to be used in a variety of ways depending on the learning needs of the students. Some suggestions are:

- As lesson starters presented on an overhead projector.
- As a supplement or extension to work being taught.
- As the basis for cooperative learning activities in the classroom.
- As an incentive for fast finishers in the class.
- As challenging homework assignments.
- As a “fill in” for the last five minutes of a lesson.
- As the basis for a school competition (e.g. “maths puzzle of the week”).

At the end of the day, how you use this resource is up to you. However, the underlying objective should always be to increase participation and enthusiasm in mathematics. In particular, I hope that you will share in my goal of teaching students to become confident mathematical problem solvers who have taken on an “I can do” attitude.



# Money Madness

An **exchange rate** is the number of units of one nation's currency that is needed to purchase one unit of another nation's currency. Exchange rates can fluctuate on a daily basis and are usually reported in newspapers or on television news programmes.



<http://www.x-rates.com>



## Problem

Ngaire is a New Zealander who is planning an overseas trip. She will spend four nights in Melbourne, Australia, before heading to Tokyo for ten days.

Prior to leaving New Zealand, she visits a bank and purchases 270 Australian dollars. This costs her \$300 (NZD). She then purchases 252,000 Japanese Yen at this exchange rate  $\$1 \text{ (NZD)} = \text{¥}73 \text{ (JPY)}$ . Towards the end of her trip to Melbourne, Ngaire decides to convert some of her unspent Australian money into Japanese Yen.

If the exchange rates haven't changed since Ngaire left New Zealand, how many Japanese Yen can Ngaire expect to get for 50 Australian dollars?





# Wonderful One Hundred

Why do you think the Roman numeral for 100 is the capital letter C? Here are some clues...

There are 100 centimetres in a metre and 100 cents in one dollar.

The name centipede means 100 feet, although most centipedes have fewer feet than this.

100 years is a century and a person who lives to be 100 years old is known as a centenarian.



## Problem

Fiona found that by adding just these numbers: 1, 2, 3 and 5, she could make all the numbers from 1 to 10 inclusive.

For example:  $1 = 1$ ,  $2 = 2$ ,  $3 = 3$ ,  $1 + 3 = 4$ ,  $2 + 3 = 5$ ,  $1 + 5 = 6$ ,  $2 + 5 = 7$ ,  $3 + 5 = 8$ ,  $1 + 3 + 5 = 9$ ,  $2 + 3 + 5 = 10$ .

What is the least number of different numbers needed in order to form all the numbers from 1 to 100 inclusive? What are these numbers?

**Note:** *a number cannot be used more than once in any one addition.*



## Reading Rapture

The biggest library in the world is the Library of Congress found in Washington DC, USA. The library employs about 4600 people and is home to almost 119 million items. If the bookshelves were laid out end to end, they would stretch to approximately 853 kilometres!



<http://www.guinnessworldrecords.com/index.asp?id=49928>



### Problem

Gary went to the library to borrow a book. He read one third of the book while he was there, one sixth of what was left when he got home, and one ninth of what remained before he went to bed. In the morning he saw that he had 120 pages left to read.

How many pages are in the book?



# Grandad's Garden

Perhaps somewhat surprisingly, the tomato is thought to be the world's most popular fruit (not vegetable!) followed by bananas and then apples. The scientific name for the tomato is *Lycopersicon lycopersicum*, which means 'wolf peach'. The tomato has also been referred to as 'the apple of love' (French) or 'the apple of paradise' (German).



<http://www.didyouknow.cd/tomatoes.htm>



## Problem

Grandad takes great pride in his vegetable garden, growing silver beet, rhubarb and tomatoes. One day Grandad went to the local garden centre and purchased 60 seedlings. Later that day he planted half of his tomato plants, one third of his rhubarb and three quarters of his silver beet before heading inside for a cup of tea.

If Grandad planted 12 silver beet plants and 10 rhubarb plants, how many tomato plants did he plant?





## Banking Blues

The earliest hint of a banking system was in Ancient Mesopotamia between 3000 and 2000 BC. During this time, the royal palaces and temples became safe places for storing grains and other valuables. Early forms of money included precious metals, shells, rice, dog's teeth, metal disks, spades, hoes and knives.



<http://www.ex.ac.uk/~RDavies/arian/origins.html>



### Problem

Kathleen is desperately trying to get out of bank overdraft. Every day she manages to save \$8, however, at the end of each week the bank charges her \$25 in bank fees. If Kathleen is currently \$200 in overdraft, how long before she first gets out of overdraft?



# BEDMAS on the Brain

Here is a song to help you remember BEDMAS (sung to the tune of "The Hokey Pokey"): 'you do the brackets first, then the exponents take flight. Next you divide and multiply in order left to right. You add 'em and subtract 'em at the end and then you shout. "That's what BEDMAS is all about!"'



<http://educ.queensu.ca/~fmc/november2002/BEDMAS.html>

$$\frac{1}{2} (33 - 42) \div 8 \times 73 + 1 = ?$$

## Problem

Make the numbers 1 to 12 inclusive using only these three numbers: 1, 4 and 6.

For example:  $6 - (4 + 1) = 1$  etc.

You must use each of the numbers 1, 4 and 6 once and only once in every calculation. Use any mathematical symbols you like and remember BEDMAS!

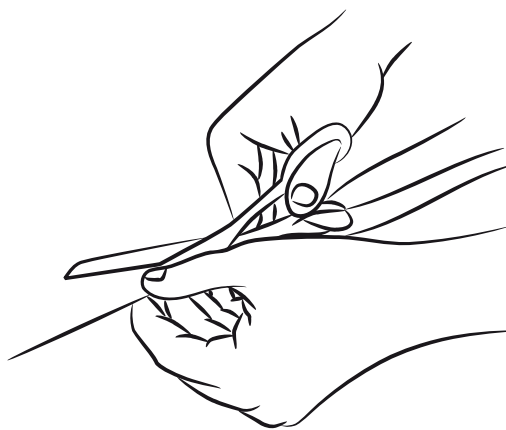


# Mighty Metres

Measurement was originally based on the human body. For example measurements were in 'feet', 'hands', the 'cubit' (from the elbow to the tip of the middle finger) or the 'span' (distance between the tip of the thumb and little finger when extended). The problem with this system was that measurements varied from person to person! Thankfully measurement is now standardised. Many countries use the metric system, where one unit of length is defined by the metre.



<http://msl.irl.cri.nz/si-units/length/index.html#The%20Metre>



## Problem

Pauline went to the store and purchased some fabric for a vest she was making.

From the clues can you work out the cost per metre of the fabric?

- The cost per metre was a two digit decimal number ending in nine.
- Pauline purchased between three and four metres of material.
- The total cost of the material was \$38.61.





# Costing the Car

According to the Guinness Book of Records, the longest car ever built was 30.5 metres long! The car has 26 wheels, a swimming pool, diving board and a king sized waterbed! The car, built by an American man, is mainly used in films or in exhibitions.



<http://www.guinnessworldrecords.com/>



## Problem

Bertha is desperately trying to sell her car. After three weeks she reduces the selling price by 30%. Two weeks later, Bertha's son Marcus offers to buy the car. As Bertha will be selling her car to a close family member, she offers to reduce the discounted price by a further 10%.

However Marcus says that he will only buy the vehicle if Bertha drops her original selling price by 40%.

Bertha replies, "That is what I am offering to do!"

"No it's not!" Marcus retorts.

Who has got their sums wrong?



# Even Odder

**Even** numbers are completely or exactly divisible by two. Numbers which are not even are called **odd** numbers.

The sum of two even numbers or two odd numbers is always an even number. Justify the following statement:

The sum of an even number and an odd number is always an odd number.

1 2 3 4 5 6 7 8 9 10

## Problem

Replace each different letter in the addition problem below with a different number to show that two odd numbers added together will result in an even number. What number does each letter represent? How many possible solutions are there?

$$\begin{array}{r} \text{ODD} \\ + \text{ODD} \\ \hline \text{EVEN} \end{array}$$



# Daylight Delight

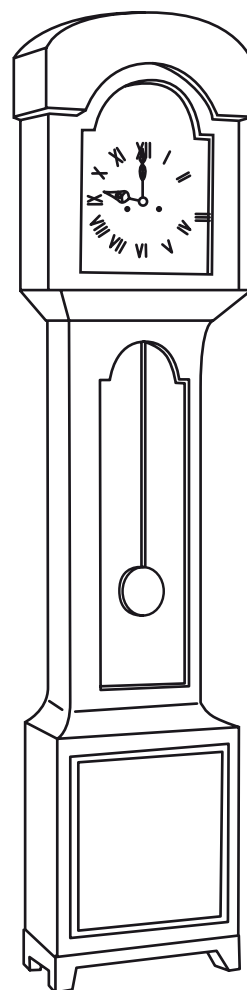
Some countries adjust their clocks to make better use of daylight. This is called **daylight saving**. Daylight saving begins in Spring when the clocks are put forward one hour, essentially moving an hour of daylight from the morning to the evening. Daylight saving finishes in Autumn when the clocks are moved back an hour. An easy way to remember this is "spring forward / fall back".



<http://webexhibits.org/daylightsaving/b.html>

## Problem

Eleven hours ago it was twelve hours since Karen went to bed. She slept for a total of nine hours. If today is the first day of daylight saving and the time now is 2100, what time did Karen get up this morning?





# Pool Perplexity

Belgium is home to the world's deepest swimming pool. Opened in 2004, the pool is 33 metres deep and took seven years to create! If anyone makes it to the bottom they will encounter a number of different passageways and rooms. The pool is not intended for racing but instead for scientific research and for learning how to dive.



[http://news.bbc.co.uk/cbbcnews/hi/world/newsid\\_3771000/3771073.stm](http://news.bbc.co.uk/cbbcnews/hi/world/newsid_3771000/3771073.stm)



## Problem

Chris sees a circular swimming pool advertised for sale in the local paper and wonders whether it will fit into his backyard. Chris measures his yard only to find that it is perfectly square with sides 5.5 metres long.

The advertisement states that the pool has an area of  $28 \text{ m}^2$ . Will the pool fit into Chris's yard?

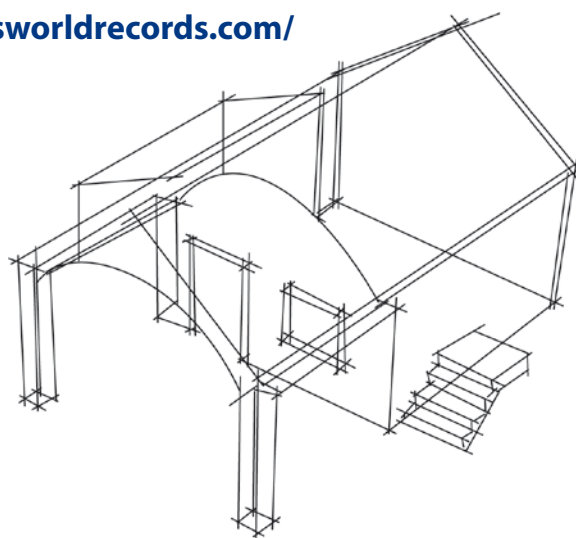


# Bedroom Bedlam

Beginning in 1886, Winchester House in San Jose, California, USA took 38 years to be transformed from an 8-room farmhouse to a mansion comprising 13 bathrooms, 10000 windows, 40 staircases, 2000 doorways and 47 fireplaces! The house is also referred to as the Winchester Mystery House due to strange characteristics such as staircases leading nowhere, a window in the floor and closets opening to blank walls.

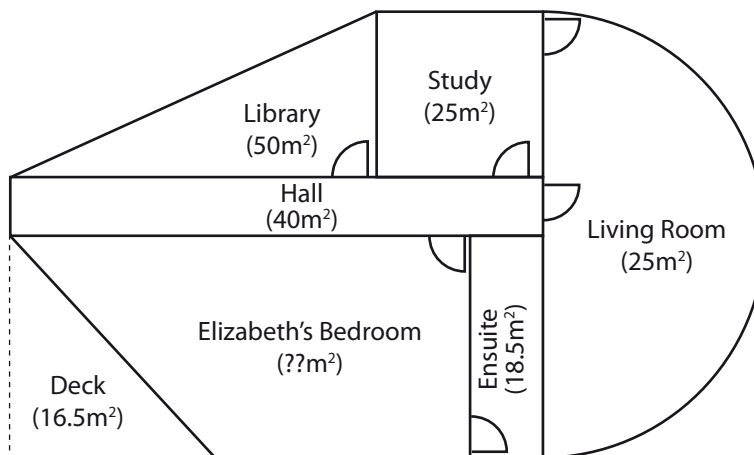


<http://www.guinnessworldrecords.com/>



## Problem

Elizabeth is really excited, having just seen the plans for the bottom storey of her new dream home. She can't wait to order new carpet but notices that the builders have accidentally left off the area measurement for her bedroom. Given that she knows the study is perfectly square and the living room is semi-circular, can you help Elizabeth calculate the floor area of her bedroom?





# One Wheel Wonder

To ride a unicycle an excellent sense of balance is needed because, as the name suggests, a unicycle consists of just one wheel. Unicycles are often associated with clowns at the circus, however some people ride unicycles as a sport. Every two years, the best unicyclists in the world contest events such as mountain unicycling, unicycle trials and unicycle hockey at the UNICON world championships.



<http://en.wikipedia.org/wiki/Unicycle>



## Problem

A clown on a unicycle rides the circus high wire at a rate of 15 revolutions per minute. This takes him 6 minutes. To make the return trip, he jumps on a second unicycle with a wheel that is half the height of the first unicycle's wheel. If the clown wishes to make the return trip in half the time that it took him to get across, how many revolutions per minute should he make?



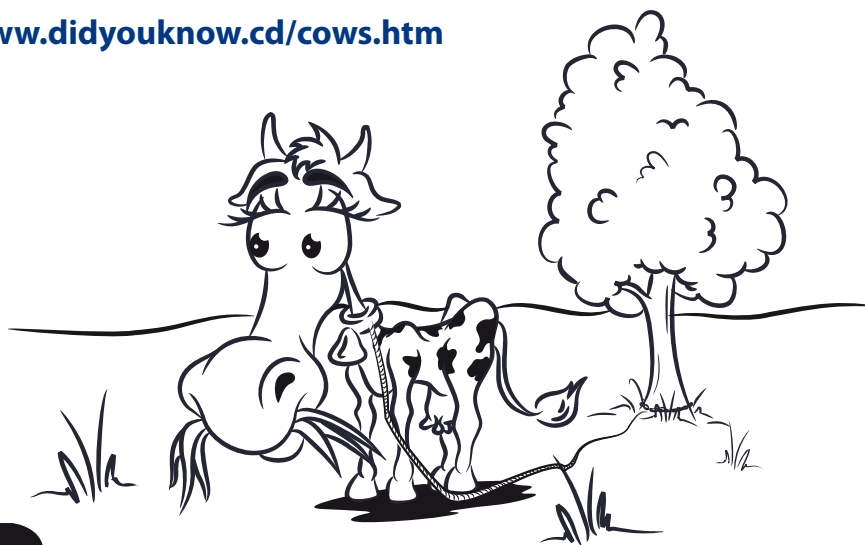


## Cud for the Cow

It is just as well cows have four stomachs because they spend about eight hours of every day eating! Despite having no upper front teeth, cows consume about 45 kg of feed and drink enough water to fill a bathtub on any given day. Hydrocarbon emissions from cows, mainly brought about by burping, are considered a major cause of the hole in the ozone layer!



<http://www.didyouknow.cd/cows.htm>



### Problem

A lone tree, situated in the centre of a large paddock, has a circular trunk with a radius of 0.5m. A chain is tied around the trunk. A rope is fixed to the chain and Cocoa the cow is attached to the rope. Cocoa is free to move around the tree, as the chain will swivel freely around the tree's trunk.

If the rope that Cocoa is attached to is 6 metres long, what is the total area of grass that Cocoa can reach.



# Thinking Outside the Square

Have you heard the following claim? It's impossible to fold a piece of paper (no matter how big or small) in half more than seven or eight times. Britney Gallivan was the first person to disprove this theory. Britney showed that it was possible to fold an \$85 roll of toilet paper, nearly 4000 feet long, in half 12 times!



[http://csmp.ucop.edu/cmp/comet/2004/09\\_03\\_2004.html](http://csmp.ucop.edu/cmp/comet/2004/09_03_2004.html)

## Problem

I fold the four corners of a perfectly square piece of paper into the centre so that my square is now double its original thickness.

If the original square measured 20 cm x 20 cm, what is the length of each side of the new square?



# Tremendous Travels

Greenwich Mean Time (GMT) marks official time around the world. GMT is measured from the Greenwich Meridian Line in England, which is the starting point for all 24 time zones in the world. When you travel west, you put your watch back one hour for every time zone you cross. Conversely, when you travel east, you put your watch forward one hour for every time zone crossed.



<http://www.greenwichmeantime.com/>



## Problem

Three siblings, Stephen, Thomas and Anastasia are planning to meet in Sydney Australia for Christmas. Stephen will fly out from Paris at 1300 on the 23<sup>rd</sup> of December (Paris time). His flight will take 20 hours. His brother Thomas will fly from Washington DC twelve hours earlier on a 30 hour flight. Their sister, Anastasia plans to leave Wellington on the 24<sup>th</sup> of December at the same time of day as Thomas left Washington. Her flight will take 4 hours.

Given that when it is midday in Sydney it is 3 am in Paris, 2 pm in Wellington and 9 pm the previous night in Washington DC, who will arrive first in Sydney?

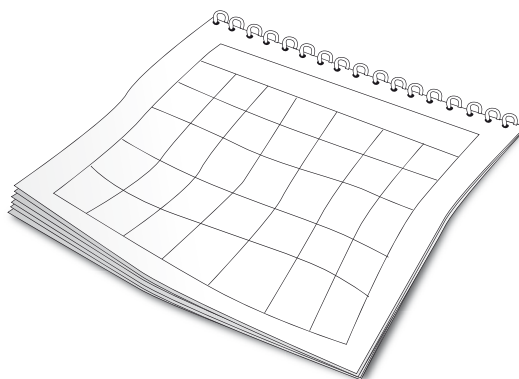


# Lots a Leaps

During a leap year, an extra day (February 29<sup>th</sup>) is added so that the calendar year has 366 days. This is to remedy the problem of the solar year being slightly longer than the calendar year. Many years ago English law essentially ignored (or 'leaped over') leap year day and so it was assumed that traditions also did not apply. Consequently women who feared they would not be proposed to, 'popped the question' on leap year day.



<http://newsinjection.ihug.co.nz/Feb04/leap.htm>



## Problem

A leap year is any year that is divisible by four. There is one exception to this rule – century years (for example ...1700, 1800, 1900...) that are divisible by four but not divisible by 400 are not considered leap years. Using this information, how many seconds will tick by in the 21<sup>st</sup> century?



# Marathon Mania

The smallest country in the world is the Vatican City, located entirely within the city of Rome, Italy. The country is the headquarters of the Roman Catholic Church and consists mostly of clergy. The area of the country is just  $0.44 \text{ km}^2$  and it has a perimeter of  $3.2 \text{ km}$ .



<http://www.brainsluice.com/miscellanea/ickle/archive/vaticancity.html>



## Problem

Brenda and Geoff are top marathon runners who train for an upcoming event by running the  $3.2 \text{ km}$  perimeter of the Vatican City. They both start at the same place (not far from Saint Peter's Square) but run in opposite directions.

When Brenda meets Geoff she instantly turns and makes her way back to the starting point. When she reaches the starting point she turns instantly again and heads back towards Geoff, while he keeps running as before. She continuously makes these journeys from the starting point to Geoff until he completes one lap.

Geoff's speed is  $12 \text{ km per hour}$  and Brenda's speed is  $15 \text{ km per hour}$ . How far does Brenda run?



# Pyramid Predicament

The ancient pyramids of Egypt continue to be a popular tourist attraction. The largest pyramid is the great pyramid at Giza, containing about 2,300,000 blocks of stone weighing an average of 2.5 tonnes each! Much controversy still surrounds how the pyramids could have been constructed without the help of modern technology.

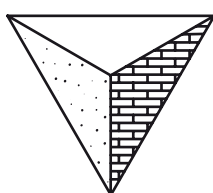


<http://homepage.powerup.com.au/~ancient/pyra1.htm>

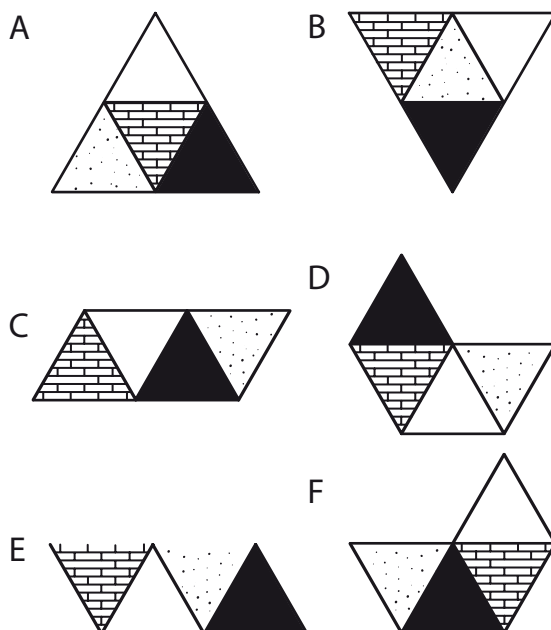


## Problem

Tanya constructed a regular pyramid using paper. The bird's eye view of her pyramid is shown.



Each face is an equilateral triangle showing a different pattern (white, bricks, dots and black). Of the six nets shown below, which one could be folded to give Tanya's pyramid?







# Photocopying Fiasco

Can you imagine life before the photocopying machine? Secretaries had to create multiple copies using carbon paper and students had to take notes from reference books by hand. Thank goodness for Chester F. Carlson's 1937 invention of the photocopier.



## Problem

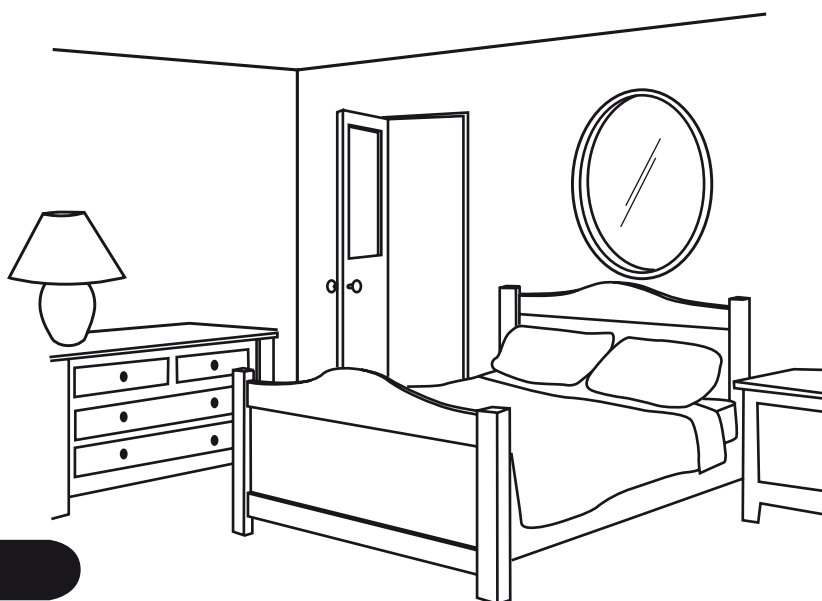
Heather wishes to enlarge a photograph of her family to 1.5 times the size of the original copy. She uses a coloured photocopier and types in "1.5". Later Heather decides that she wants another photocopy - this time a reduction of the original photograph. However, having discarded the original, Heather only has the enlarged copy to work with.

What should Heather type in to the photocopier in order to reduce the enlarged photograph to one third of the size of the original?



# Mirror, Mirror on the Wall

Have you ever wondered why so many people hate photographs of themselves? Perhaps it is because people are used to seeing their reflection where the left side of their face appears on the right and vice versa. How the rest of the world sees you may be slightly different to how you see yourself (unless of course your face is truly symmetrical).



## Problem

When I look into the mirror in my bedroom, I see the room as above.

Suppose I now turn away from the mirror and look at my bedroom through my own eyes. Which of the statements below is false?

- A. If I were to look into my bedroom through the window above my bed, the big chest of drawers would be on the left.
- B. The lamp is on the right side of the big chest of drawers.
- C. If I were to walk through the door of my bedroom, my bed would be on my left.
- D. When I lie in my bed at night, my small bedside table is to my right.

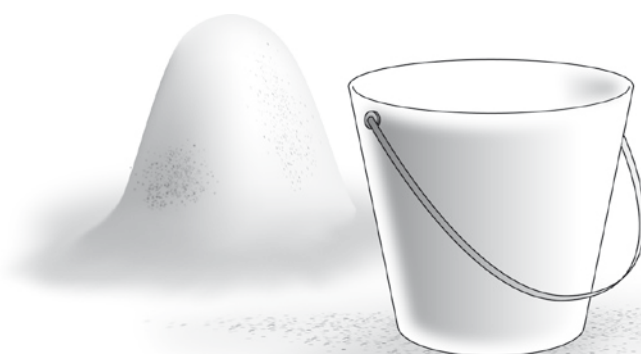


## A Sandy Situation

Who has ever heard of green sand? On the southern tip of the big island of Hawaii, an almost mythical green sand beach exists. The sand is comprised of volcanic glass that is tinted green by the presence of olivine crystals. With an average size of 2 mm, the green sand is considered very coarse.

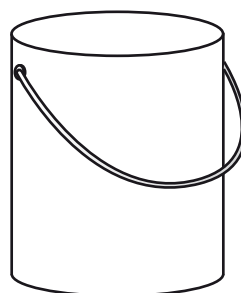
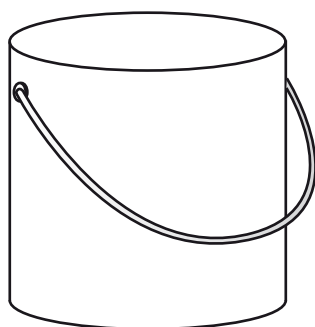


<http://www.frenchspot.com/Hawaiian/Photos/Greensand/greensand.html>



### Problem

Holly and Renee were building green sand castles at the beach using identically shaped cylindrical buckets.



To make the castles, the girls filled their buckets with wet sand, then carefully turned them upside-down and emptied the contents. On measuring their castles, Holly discovered that hers was 30 cm tall. Renee found that her castle was 15 cm taller and 1.5 times wider than Holly's.

If the Holly's bucket can hold 3000 cubic centimetres of wet sand, how much sand was used in Renee's sandcastle?



# Logo Logistics

Logos are visual images comprising a combination of letters, symbols or other graphics. On July 24, 1999, a world record was broken when 34,309 people gathered to form the largest human logo. The logo created was the Portuguese Euro 2004 logo and was part of the nation's bid to host a soccer tournament.

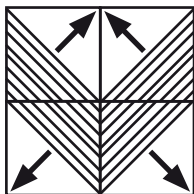


[http://www.guinnessworldrecords.com/content\\_pages/record.asp?recordid=48857](http://www.guinnessworldrecords.com/content_pages/record.asp?recordid=48857)



## Problem

Susie has designed a logo for a new business venture:



However, having sent the design to the printing company, she suddenly changes her mind. She faxes the following instructions through to the company:

Rotate the image  $270^\circ$  about the centre.

Reflect the resulting image through the diagonal that runs from the bottom left hand corner to the top right hand corner.

What does Susie's logo look like now?

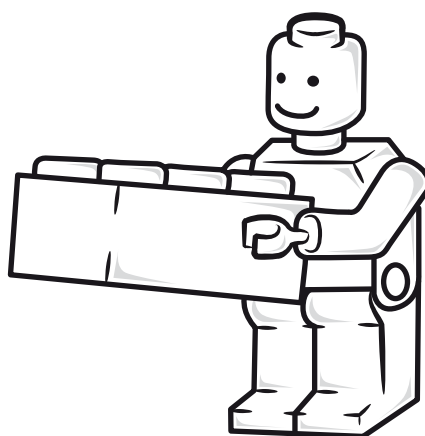


# Lego Land

In 1968 a toy maker named Ole Kirk Christiansen opened the first Lego Land in Billund, Denmark. The park offered numerous theme park rides and exhibits made purely from Lego. Four countries in the world now have Lego Lands. They are Denmark, the United States of America, Germany and England.



<http://www.lego.com/legoland/portal/default.asp?locale=2057>

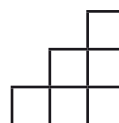


## Problem

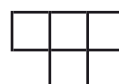
Hamish built a Lego house.  
From the side it looked like this:



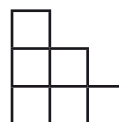
From the front it looked like this:



From the top it looked like this:

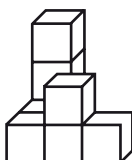


From the back it looked like this:

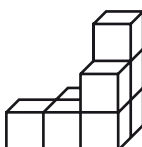


Which of the following could be Hamish's Lego house?

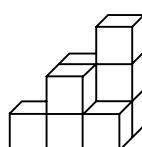
A



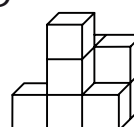
B



C



D





# Navigational Nightmare

A compass is a special device that allows people to gauge what direction they are headed. A useful mnemonic for remembering the order of the four main directions (North, South, East and West) is “**N**ever **E**at **S**oggy **W**heetbix”. Can you make up your own mnemonic?

My mnemonic is:

N \_\_\_\_\_

E \_\_\_\_\_

S \_\_\_\_\_

W \_\_\_\_\_



## Problem

To walk from the supermarket to her home, Alison must walk 400 m East, 200 m North, 800 m West, and then 500 m South. To walk from school to home, she walks 300 m East, 600 m South, 300 m East again, then 300 m North.

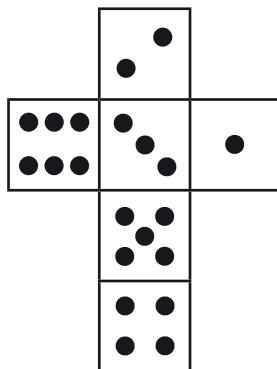
Assuming Alison can walk in a straight line from the supermarket to school, how far and in what direction would she travel?





# Distinctly Dicey

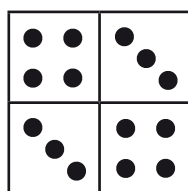
The net of a dice looks like this:



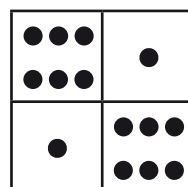
When folded, opposite sides add together to make seven.

## Problem

I fit eight dice together so as to form a  $2 \times 2 \times 2$  cube. From the front and from the back my cube looks like this:



From both sides my cube looks like this:

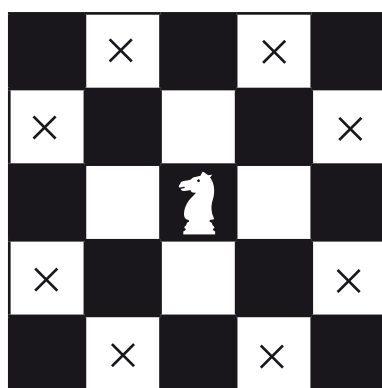


What does the top view of my cube look like?



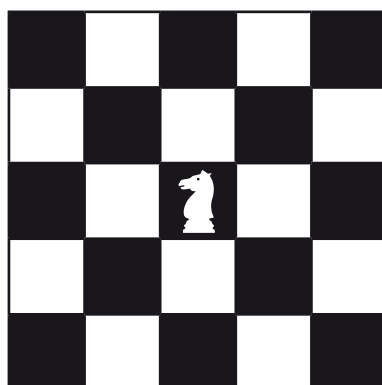
# Chess Challenge

On a chessboard there are nine different types of pieces. They are: pawn, rook, knight, bishop, queen and king. Each piece has a specific way in which it is allowed to move around the board. The movement of the knight is particularly interesting as it may only move in an L shape as shown (to any cross):

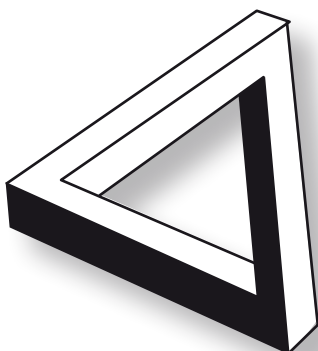


## Problem

Starting from the centre square of a 5 x 5 chessboard as shown above, move the knight to all twenty-five squares. You may not visit a square more than once. (Note that this is not a full sized chessboard!)



# creative Maths Problems - Solutions





## Money Madness

Students should demonstrate a conceptual understanding of the function of multiplication and division.

*How many Japanese Yen can Ngaire expect to get for 50 Australian dollars?*

**Solution:** 4055.56 Japanese Yen

Ngaire purchases 270 Australian dollars. This costs her \$300 New Zealand dollars. The exchange rate is therefore  $\$1 \text{ (NZD)} = \$0.9 \text{ (AUS)}$ .

$\$1 \text{ (NZD)} = 73 \text{ (JPY)}$ . Thus,  $\$0.9 \text{ (AUS)} = 73 \text{ (JPY)}$ . This means that  $\$1 \text{ (AUS)} = 81.1111 \text{ (JPY)}$

Ngaire exchanges \$50 (AUS). Therefore she gets  $50 \times 81.1111 = 4055.56$  Japanese Yen.

### Suggestions for Teachers

**Lead with questions:** What information do you need in order to answer the question? How can you get this information? Explain in your own words how to determine the New Zealand – Australian exchange rate. Why does this method work?

**Make it concrete:** Involve students in a simple role-play, using pretend money to strengthen understanding.

**Work together:** This is a problem that could be worked through in pairs to encourage further discussion and debate.

**Investigate further:** What effect will premature rounding have on the solution?

**Further activities:** Use this problem as an opening to further discussion, investigation or activities involving current exchange rates in local papers or television. Ask students to repeat the problem using the “real” information.



## Wonderful One Hundred

Students should demonstrate number and computation sense.

*What is the least number of different numbers needed in order to form all the numbers from 1 to 100 inclusive? What are these numbers?*

**Solution: 8 numbers:** 1, 2, 3, 5, 10, 20, 30, 50

The numbers 10, 20, 30 and 50 will form all the multiples of ten between 1 and 100 inclusive. The numbers 1, 2, 3 and 5 are also required in order to form those numbers that are not multiples of ten.

This amounts to eight numbers in total.

### Suggestions for Teachers

**Point the way:** Inevitably, some students will fail to see that there is a relationship between Fiona's findings and the solution to the problem. Hint that there is a connection but encourage students to make and justify the link for themselves.

**Consolidate:** Discuss the solution as a group. Reinforce by asking students to briefly write down an explanation in their own words. Swap with a friend.

**Investigate further:** Is there more than one solution to this problem? What is the minimum number of numbers needed to make one thousand, ten thousand or one million?

**Further activities:** (For fast finishers) Find as many different numbers as you can that will add together to make the number 100? (No negative numbers allowed).



## Reading Rapture

Students should develop an understanding of quantities expressed as fractions of a whole.

*How many pages in the book?*

**Solution:** 243 pages

Gary reads  $\frac{1}{3}$  of the book at the library. He then reads  $\frac{1}{6}$  of two thirds at home. He reads  $\frac{1}{9}$  of five ninths before he goes to bed.

Gary has read  $\frac{1}{3} + \frac{1}{9} + \frac{5}{81} = \frac{41}{81}$  of his book so  $\frac{40}{81}$  of the book remains to be read.

As 120 pages equates to  $\frac{40}{81}$  of the book, the total number of pages in the book must be 243.

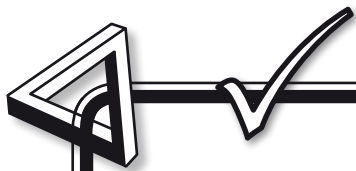
### Suggestions for Teachers

**Break it up:** This problem has the potential to cause confusion. Recommend that students break it up – bit by bit. A table might be a useful way for organising the information.

**No calculators:** This problem provides practice at finding equivalent fractions without too much difficulty.

**Point the way:** Expect that some students will calculate how much of the book Gary has read but fail to calculate what fraction remains. Refer students back to the original question.

**Explain further:** Some students may have difficulty accepting that  $\frac{120}{x} = \frac{40}{81}$  where  $x$  = total number of pages in the book. This point may need to be explained further using the board.



## Grandad's Garden

Students should find fractions equivalent to the one given.

*How many tomato plants did Grandad plant?*

**Solution:** 7

Grandad planted 12 silver beet plants, which is three quarters of the total silver beet plants purchased. Therefore Grandad bought 16 silver beet plants.

He planted 10 rhubarb plants, which was  $\frac{1}{3}$  of the total number of rhubarb plants. Therefore Grandad bought 30 rhubarb plants.

As Grandad purchased 60 plants altogether, he must have bought 14 tomato plants. He planted half of these, which equates to 7.

### Suggestions for Teachers

**Break it up:** Encourage students to consider each piece of information separately. Perhaps write on the board: How many silver beet plants did Grandad purchase? How many rhubarb plants did Grandad purchase? How many tomato plants did Grandad purchase?

**Work together:** After a solution has been found, ask students why it was necessary to know how many of each plant Grandad had purchased in order to answer the question. Students could spend 1-2 minutes in pairs clarifying this.

**No calculators:** The numbers and fractions are basic and calculators should not be needed.





## Banking Blues

Students should perform calculations involving negative numbers.

*When will Kathleen first get out of overdraft?*

**Solution:** On the sixth day of the sixth week.

By the end of the first week, Kathleen has saved \$56 bringing her balance to -\$144, however the bank charges her fees of \$25. Therefore she starts her second week with a bank balance of -\$169, which is a total increase of \$31.

At the end of five weeks, Kathleen will have increased her balance by  $\$31 \times 5 = \$155$  bringing her total to -\$45. On the sixth day of the sixth week, Kathleen will have saved a further \$48 ( $\$8 \times 6$ ) and so her account will be in credit by \$3.

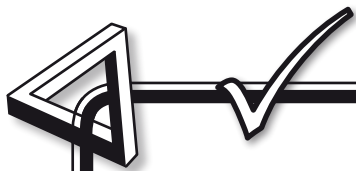
Hence the sixth day of the sixth week will be the first occasion when Kathleen gets out of overdraft.

### Suggestions for Teachers

**No calculators:** One key aim of this problem is to give students contextual understanding for negative numbers. Performing the calculations without the help of a calculator should help to strengthen understanding and provide good practice.

**Break it up:** Many students may employ a rather long and complicated approach, leading to a lot of confusion. Suggest forming a table to document Kathleen's bank balance at the end of each week and at the end of each day during the sixth week.

**Make it concrete:** Some students will almost certainly answer 'after seven weeks', their justification being that  $\$31 \times 7 = \$217$  which is greater than \$200. This of course is incorrect. Set up a simple role-play using pretend money to demonstrate what happens during the sixth week. Also stress the importance of answering questions as precisely as possible.



## BEDMAS on the Brain

Students should demonstrate knowledge of the conventions for order of operations.

*Can you make the numbers 1 to 12 inclusive using just these numbers: 1, 4 and 6?*

### Solution:

One possible solution:

$$6 - (4 + 1) = 1$$

$$6 \times 1 - 4 = 2$$

$$6 - 4 + 1 = 3$$

$$16 / 4 = 4$$

$$16 - 1 = 5$$

$$6 / 1 = 6$$

$$6 - 1 + \sqrt{4} = 7$$

$$14 - 6 = 8$$

$$6 + (4 - 1) = 9$$

$$6 \times 1 + 4 = 10$$

$$6 + 4 + 1 = 11$$

$$\sqrt{4} \times 6 / 1 = 12$$

### Suggestions for Teachers

**Work together:** Divide the class into teams and compete to see which team can find a solution in the shortest time. Give extra points for originality and creativity!

**No calculators:** The purpose of this problem is to familiarise students with BEDMAS and provide practice at mental arithmetic.

**Investigate further:** Provide further challenges for fast finishers. For example: carry on to make the numbers from 13 to 20 inclusive or can you make the numbers from 1 to 12 inclusive using the numbers 2, 3 and 6?

**Discuss:** Why do you think it necessary for mathematicians to follow BEDMAS?



## Mighty Metres

Students should solve a problem involving decimal multiplication and division.

*What is the cost per metre of the fabric?*

**Solution:** \$9.90

In metres, the length of fabric bought must be a two digit decimal number beginning with three. A one appears in the hundredths column for the total cost of the fabric (\$38.61). As the cost per metre is a two digit decimal number ending in nine, the length of fabric purchased must also end in 9 (as  $9 \times 9$  produces a number ending in one).

Therefore the total amount of fabric bought was 3.9 metres. Dividing \$38.61 by 3.9 gives a cost per metre of \$9.90.

### Suggestions for Teachers

**Point the way:** Suggest setting the problem in the usual way for carrying out a decimal multiplication. Replace unknown numbers with empty boxes.

**No calculators:** The purpose of the problem is to strengthen understanding for decimal multiplication and division. Calculators will encourage students to utilise a "guess and check" method.



## Costing the Car

Students should explore the use of percentages in an everyday context.

*Who has got their sums wrong?*

**Solution:** Bertha

Let the original selling price of the vehicle be represented by 'C'.

Bertha reduces the original selling price by 30% so the discounted price is  $0.7 \times C$ .

Reducing the discounted price by a further 10% means that the vehicle is now being sold for  $0.9 \times 0.7 \times C = 0.63 \times C$ .

Therefore Bertha has reduced the original selling price by 37% not 40%.

### Suggestions for Teachers

**Lead with questions:** Suppose the original selling price of the car is \$1000. What would it cost after the various discounts? Can you generalise your findings?

**Discuss:** Where did the other 3% go? Explain. Is reducing the price first by 30% and then by a further 10% the same as reducing the price first by 10% and then by 30%? Why or why not?



## Even Odder

Students should demonstrate familiarity with the procedure for summing two whole numbers.

*What number does each letter represent?*

**Solution:** Two possible solutions:

$$O = 8$$

$$D = 5$$

$$E = 1$$

$$V = 7$$

$$N = 0$$

or

$$O = 6$$

$$D = 5$$

$$E = 1$$

$$V = 3$$

$$N = 0$$

As all letters must represent a different number, D has to be five or more. However E has to be one because the sum must be a number less than 2000. Therefore D must be five, making  $N = O$  and  $E = 1$ .

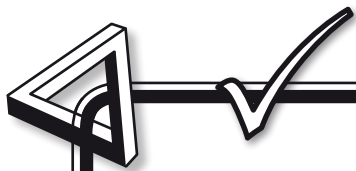
O must be five or more (otherwise the sum would be less than 1000). However O is not five (as  $D = 5$ ). O can also not be seven or nine. Therefore O is either equal to six or eight, making V equal to three or seven respectively. There are therefore two possible solutions.

### Suggestions for Teachers

**Lead with questions:** Could D be a number less than five? Why or why not?  
Could E possibly be a number greater than 1? Why or why not?

**Point the way:** If students are having difficulty, work through some (incorrect) possibilities for the letter D on the board. Ask for class input.

**Further activities:** Challenge students to invent their own similar problem.



## Daylight Delight

Students should perform calculations involving time, including 24 hour clock time.

*What time did Karen get up this morning?*

**Solution:** 0700 or 7 am

The time now is 2100 (or 9 pm). Eleven hours ago it was 1000 (or 10 am) daylight saving time. As Karen went to bed 12 hours before 1000, she went to bed at 9 pm (an hour has been lost due to daylight saving).

Karen had a total of nine hours sleep. Adding nine hours to 2100 (or 9 pm) gives 0600 (or 6 am). But it is daylight saving and the clocks are put forward one hour. Therefore Karen woke up at 0700 (or 7 am).

### Suggestions for Teachers

**Point the way:** Ensure students know when clocks are officially turned forward to mark the start of daylight saving.

**Break it up:** Students should consider each piece of information separately and systematically.

**Make it concrete:** It will be helpful to actually demonstrate this problem using a clock.



## Pool Perplexity

Students should demonstrate a thorough understanding of the formula for the area of a circle.

*Will the pool fit into Chris's yard?*

**Solution:** No

The pool is circular with area  $28 \text{ m}^2$ , thus the radius of the pool is 2.99 m.

As Chris's backyard is square with sides 5.5 m long, it will not fit a swimming pool with diameter 5.97 m.

### Suggestions for Teachers

**Draw a diagram:** Expect that some students will incorrectly conclude that the circular pool will fit because the area of the pool is less than the area of the garden. Suggest students check their solution by drawing a scale diagram of the problem.

**Consolidate:** The tricky aspect is rearranging the formula for the area of the circle in order to calculate the radius. Students may use 'trial and error' instead. Don't discourage this, as it should help to consolidate the area formula.

**Point the way:** If and when appropriate, give hints regarding the easier algebraic method for calculating the radius.





## Bedroom Bedlam

Students should perform a series of calculations using the formulas for the area of a square, rectangle, triangle and circle.

*What is the floor area of Elizabeth's new bedroom?*

**Solution:**  $150 \text{ m}^2$

The study is  $25 \text{ m}^2$  therefore the sides of the study must be 5 m long. The library, being a right-angled triangle, must also have side length = 5 m. As the library is  $50 \text{ m}^2$  the base length of the room is 20 metres. Therefore the hallway is 25 m long. As the hallway is  $40 \text{ m}^2$ , its width must be 1.6 m.

The living area is  $77 \text{ m}^2$  and is semicircular. The radius is therefore 7 m and so the diameter is 14 m. As the study is 5 m and the hallway is 1.6 m, the ensuite must be 7.4 m long. Given that the ensuite has an area of  $18.5 \text{ m}^2$ , its width must be 2.5 m.

The hallway is 25 m long and the ensuite is 2.5 m wide, making Elizabeth's bedroom 22.5 m long. The area of Elizabeth's bedroom is therefore  $22.5 \times 7.4 - 16.5$  (deck area) =  $150 \text{ m}^2$

### Suggestions for Teachers

**Lead with questions:** What measurements do we require in order to calculate the area of the bedroom? How can we obtain these measurements? Where would a sensible starting place be?

**Draw a diagram:** This problem requires students to perform a series of systematic calculations. They will almost certainly run into trouble unless they draw a simple replica of the house plan to work from.

**Point the way:** Start by asking students for formulae that might be useful. Write these on the board for students to refer to.

**Explain further:** Students are likely to require assistance or guidance in calculating the diameter of the semi-circle.



## One Wheel Wonder

Students should solve a problem by interpreting and using information about rates.

*How many revolutions per minute?*

**Solution:** 60 revolutions / minute

The wheel of the second unicycle is half the height of the first wheel and therefore has half the circumference.

In order to maintain the same speed as before, the clown would need to double the number of revolutions per minute from 15 to 30. As he wishes to halve the time it takes, he will need to double the number of revolutions per minute he makes from 30 to 60.

### Suggestions for Teachers

**Lead with questions:** How many revolutions if he wished to maintain the same speed on the way back? Does more revolutions per minute mean a faster speed? Why or why not? Is the circumference of the second wheel half the circumference of the first wheel? How do you know? How could you check?

**Explain further:** If appropriate, investigate in greater depth the claim that the circumference of the second wheel is half that of the first wheel. If students are familiar with the circumference formula, confirm using algebra.

**No calculators:** This question does not require the use of calculators

**Make it concrete:** Replicate this problem using two circular objects e.g. cans, jars, buckets etc. to represent the two unicycles. Investigate the ideas of revolutions per minute, speed and distance.



## Cud for the Cow

Students should solve a problem requiring the use of the area formula for a circle.

*What is the total area of grass that Cocoa can reach?*

**Solution:** 131.9 metres

The radius of the tree is 0.5 metres therefore the area of the tree is  $0.78 \text{ m}^2$ .

A circle with radius 6.5 m (the length of Cocoa's rope plus the radius of the tree) has an area of  $132.7 \text{ m}^2$ . Subtracting the area of the tree ( $0.78 \text{ m}^2$ ) gives 131.9 metres.

### Suggestions for Teachers

**Point the way:** Recall as a group, the area formula that is required. Write this on the board for ease of reference.

**Draw a diagram:** Suggest students draw a diagram of the situation and shade the areas they calculate.

**Discuss:** Why is the solution to this problem not overly precise? What other information is needed in order to calculate the grazing area more accurately?



## Thinking Outside the Square

Students should apply their knowledge of area and the area of a rectangle formula.

*What is the length of each side of the new square?*

**Solution:** 14.1 cm

The original square has had its thickness doubled. Therefore the area of the new square is half that of the original square i.e.  $400 / 2 = 200 \text{ cm}^2$

A square with area =  $200 \text{ cm}^2$  has sides that are 14.1 cm long.

### Suggestions for Teachers

**Lead with questions:** What do you notice about the area of the original square and the area of the new square? How could this information be of help?

**Make it concrete:** Allow students to experiment with paper squares.

**Draw a diagram:** Suggest drawing a diagram as a means of visualising how the square is to be folded.

**Investigate further:** If students are familiar with Pythagoras they could check their answer using the formula.



## Tremendous Travels

Students should be able to perform calculations involving time, including 24 hour time.

*Who will arrive first in Sydney?*

**Solution:** Thomas

Stephen leaves at 1300 on the 23<sup>rd</sup> December (Paris time). This equates to 2200 on the 23<sup>rd</sup> December (Sydney time). His flight takes 20 hours. Therefore he will arrive in Sydney at 1800 on the 24<sup>th</sup> December in Sydney.

Thomas leaves 12 hours earlier than Stephen. Therefore Thomas leaves at 1000 on the 23<sup>rd</sup> December (Sydney time). His flight takes 30 hours. Therefore he will arrive in at 1600 on the 24<sup>th</sup> December in Sydney.

Anastasia leaves at the same time of day as Thomas on the 24<sup>th</sup> of December. Thomas left at 1000 Sydney time which equates to 1900 the previous night in Washington DC. Therefore Anastasia must have left Wellington at 1900 on the 24<sup>th</sup> of December, which equates to 1700 Sydney time. Her flight takes four hours. Therefore Anastasia arrives in Sydney at 2100 on the 24<sup>th</sup> of December.

Thus Thomas arrives in Sydney first.

### Suggestions for Teachers

**Break it up:** Students will undoubtedly become confused unless they work through the information systematically, considering each of the three siblings individually.

**Lead with questions:** What time it is in Sydney when each of the three siblings departs their respective countries? Why might it make it easier to consider this?

**Further activities:** As an extra task, students could practice converting times used in this problem from 24 hour time to 12 hour time e.g. "What time did Thomas leave Washington in 12 hour clock time?", etc.

**Work together:** Students could work in groups of three to solve the problem with each student tasked with one of the three siblings. Students must explain to the others in their group how they obtained their answer.



## Lots a Leaps

Students should demonstrate an understanding of the relationship between seconds, minutes, hours, days and years.

*How many seconds will tick by during the 21st century?*

**Solution:**  $3.15576 \times 10^9$  seconds (or 3,155,760,000 seconds)

There are 25 leap years and therefore 75 non leap years in the 21<sup>st</sup> century. The leap years are as follows: 2000, 2004, 2008, 2012, 2016, 2020, 2024, 2028, 2032, 2036, 2040, 2044, 2048, 2052, 2056, 2060, 2064, 2068, 2072, 2076, 2080, 2084, 2088, 2092, 2096.

Total number of seconds in 25 leap years =  $60 \times 60 \times 24 \times 366 \times 25 = 7.9056 \times 10^8$

Total number of seconds in 75 non leap years =  $60 \times 60 \times 24 \times 365 \times 75 = 2.3652 \times 10^9$

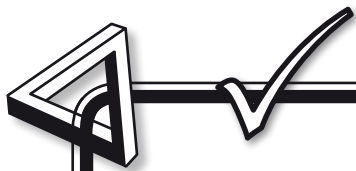
Therefore the total number of seconds in the 21<sup>st</sup> century is  $3.15576 \times 10^9$ .

### Suggestions for Teachers

**Point the way:** Start by considering how many seconds make up a minute, an hour and a day.

**Lead with questions:** Will there be more or less seconds in a leap year compared to a non leap year? How many leap years will occur during the 21<sup>st</sup> century?

**Investigate further:** Fast finishers may wish to calculate or approximate how many seconds have ticked by since they were born.



## Marathon Mania

Students should devise a strategy for solving a number problem involving rates

*How far does Brenda run?*

**Solution:** 4 km

Geoff's speed is 12 km / hour. One lap has a total distance of 3.2 km (or 3200 metres). Therefore, Geoff will complete one lap in 16 minutes.

Geoff and Brenda will be running for the same amount of time (16 minutes). As Brenda's speed is 15 km / hour, she will run  $15 / 60 \times 16 = 4$  km.

### Suggestions for Teachers

**Draw a diagram:** This may help students to visualise what is happening.

**Make it concrete:** Two students could take part in a role-play for the class to clarify the situation.

**Consolidate:** Write a written explanation, using your own language, explaining how to calculate Geoff's running time. Why does this calculation work?

**Investigate further:** Suppose Brenda and Geoff swapped speeds. How far would Brenda run now?



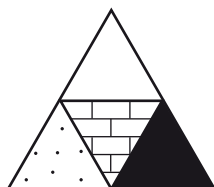


## Pyramid Predicament

Students should become familiar with the net of a tetrahedron.

*Which net could be folded to make Tanya's pyramid?*

**Solution:** Net A



Net B is eliminated because unlike Tanya's pyramid, the brick face is to the left of the dotted face.

Net C and Net D can be eliminated because the brick face is to the left of the white face. In Tanya's pyramid, the brick face is to the right of the white face.

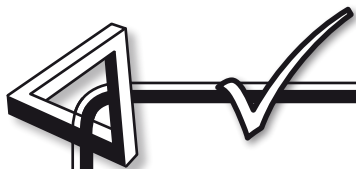
Nets E and F can be eliminated immediately because they cannot be folded to form a pyramid.

### Suggestions for Teachers

**Make it concrete:** It is understandable that many students may find this problem difficult to visualise. Suggest students actually make the nets for themselves. This will also demonstrate that nets E and F will not fold to form a pyramid.

**Work together:** If time is limited, students could work cooperatively in groups of three. Each student makes one of the three net types, colouring both sides to represent each of the six options.

**Lead with questions:** Is it necessary that all faces in Tanya's pyramid are equilateral triangles? Why or why not? What is the same and what is different about Tanya's pyramid and the pyramids of Egypt? Faster students may attempt to also make a net for an Egyptian pyramid.



## Photocopying Fiasco

Students should solve a practical problem involving enlargement and reduction of a 2-dimensional object.

*What should Heather type in to the photocopier in order to reduce the enlarged photograph to one third of the size of the original?*

**Solution:**  $2/9 = 0.222$

One third as a fraction of  $3/2$  is  $2/9$  (in terms of algebra, reduction  $\times 3/2 = 1/3$ , so reduction =  $2/9$ ). Therefore the required scale factor is  $2/9$ .

### Suggestions for Teachers

**Point the way:** Use a computer with three simple images to represent the original photo, the enlarged photo and the reduced photo. Group the objects so that they move together as their size is enlarged or reduced. Talk about scale factors and how it relates to the change in area.

**Lead with questions:** Does enlarging the photo by a scale factor of 1.5 mean that the area of the photo increases by a scale factor of 1.5? Why or why not? What does it mean? What does reducing the photo to one third of the size of the original photo actually mean? Would you expect the original photograph and the photocopied photograph to be similar shapes?

**Draw a diagram:** Suggest students draw a simple picture in their books of the original photograph and the enlarged photograph. Discuss the scale factor and the change in area.

**Make it concrete:** Enlarge and reduce a photograph on the photocopier using the scale factors from the problem as a basis for class discussion.



## Mirror, Mirror on the wall

Students should demonstrate an understanding of the properties of reflection.

*Which of the statements is false?*

**Solution:** Statement C ("If I were to walk through the door of my bedroom, my bed would be on my left").

The mirror image shows that the bed is on the left when walking through the bedroom door. Therefore, in truth, the bed must be on the right when walking through the door. Thus Statement C is false.

The mirror image of the bedroom is in opposition to statements A, B and D. Therefore these statements are all true.

### Suggestions for Teachers

**Make it concrete:** Have a mirror handy for students to use when tackling this problem or to aid understanding when going over the solution.

**Lead with questions:** What happens when we look in a mirror? Where is the mirror in this bedroom located?

**Draw a diagram:** Draw a diagram showing the mirror image plan of the bedroom (as shown in the picture), the mirror line and the 'real' plan of the bedroom.

**Investigate further:** How does a mirror work?

**Further activities:** Use a computer, scanned photographs and 'cut, copy and paste' to explore the symmetry of faces.



## A Sandy Situation

Students should explore the relationship between the scale factor for length and the scale factor for volume.

*How much sand was used in Renee's sandcastle?*

**Solution:** 10,125 cubic centimetres of sand.

Holly and Renee have identically shaped buckets. However Renee's bucket produces a sandcastle that is 45 cm tall while Holly's castle is 30 cm tall. Therefore Renee's bucket is 1.5 times the size of Holly's bucket.

Holly's bucket holds 3,000 cubic centimetres of wet sand. As Renee's bucket is 1.5 times the size of Holly's bucket, Renee's bucket will hold  $3,000 \times 1.5^3 = 10,125$  cubic centimetres of sand.

### Suggestions for Teachers

**Point the way:** Recall similar shapes. Check understanding of 'identically shaped cylindrical buckets'. Give a hint that no volume formulas are needed in order to solve this problem (although it can be solved with formulas too).

**Lead with questions:** What do we know about Renee's bucket? Is there a relationship between Renee and Holly's buckets? What is this relationship?

**Make it concrete:** Make a variety of 3-dimensional rectangular prisms from cubed blocks. Some prisms should be similar in shape and some not. Ask the students which prisms are similar and why. For those that are similar, discuss the relationship between height, surface area and volume.

**Investigate further:** Able students may like to verify their answer, working backwards using the volume formula for cylinders. Check that the radius of Renee's bucket is 1.5 times that of Holly's and that the circular base area of Renee's bucket is  $1.5^2$  that of Holly's bucket.

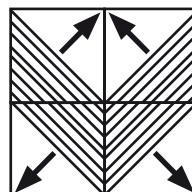


## Logo Logistics

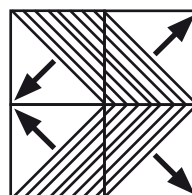
Students should successfully rotate and reflect a given object.

*What does Susie's logo look like?*

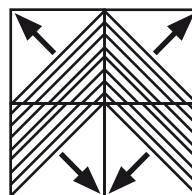
**Solution:**



Rotating the image  $270^\circ$  about the centre gives:



Reflecting this image through the diagonal that runs from the bottom left hand corner to the top right hand corner.



### Suggestions for Teachers

**Lead with questions:** What is important to remember when rotating objects? (That is, what way do we rotate objects?)

**Make it concrete:** Suggest that students replicate Susie's logo for themselves if they are having difficulty visualising the reflections and rotations in their minds.

**Initiate class discussion:** Is rotating  $270^\circ$  clockwise the same as rotating  $90^\circ$  anticlockwise? Why? How could Susie have worded the instruction to make it simpler to understand? How many lines of symmetry does this logo have? What about rotational symmetry?

**Further activities:** Students could create their own business logo with instructions and swap with a friend. Look at familiar logos of local businesses, franchises or products. Consider lines of symmetry.

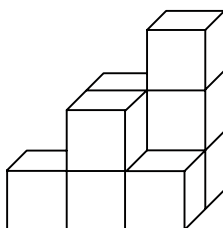


## Lego Land

Students should visualise a 3-dimensional object using views from the top, front, side and back.

*Which of the following could be Hamish's house?*

**Solution:** Model C is the only model that fits the criterion.



### Suggestions for Teachers

**Make it concrete:** Make blocks available for students to use to explore this problem. When going over the solution, present students with real models of the four different possibilities to increase understanding.

**Break it up:** Consider each model in turn. Suggest students run through all the criteria with each of the four possibilities one by one.

**Further activities:** Students could build their own model and practice drawing different 3-dimensional views of their model on isometric paper. Swap with a friend who must try and re-build the model from the drawings.



## Navigational Nightmare

Students should specify location using bearings and grid references.

*How far and in what direction would Alison travel?*

**Solution:** 1,000 m (or 1 km) West

Assume the Supermarket is at the origin (0,0). Therefore Alison's house is located at the point (-400, -300).

Using the information provided and working backwards from Alison's home, Alison's school is found at the point (-1,000, 0).

To walk from the supermarket to school, Alison must therefore walk 1,000 metres West.

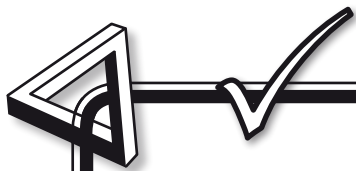
### Suggestions for Teachers

**Make it concrete:** Act out the situation using volunteers to be Alison, the supermarket, home and the school. This may help some students to better understand why it is necessary to work backwards to determine the location of the school.

**Draw a diagram:** Most students will approach this problem by drawing a simple diagram to represent the situation. Encourage this.

**Point the way:** Suggest that students utilise grid coordinates to solve the problem. Start them off on the board by showing the supermarket at the origin.



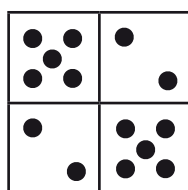


## Distinctly Dicey

Students should mentally rotate and visualise a three dimensional object.

*What does the top view of my cube look like?*

**Solution:** This is the only solution that fulfils the criterion (watch the direction of the two dotted face).



### Suggestions for Teachers

**Draw a diagram:** Students could draw a 3-dimensional diagram or a simple net of the cube to fill in as they go.

**Lead with questions:** Looking at the front of the cube - what number will be to the right of the top right hand three? What number will be to the left of the top left hand four?

**Make it concrete:** Students may find it useful to make a dice from cardboard using the net as a guideline.

**Discuss:** Talk about the rotational symmetry of the cube.

**Investigate further:** What would the bottom view of the cube look like?



## Chess Challenge

Students should plan and carry out mathematical exploration into a geometrical problem.

*Can you move the knight around all twenty-five squares without visiting each square more than once?*

Solution:

19	8	13	2	25
14	3	18	7	12
9	20	1	24	17
4	15	22	11	6
21	10	5	16	23

Explanation:

2	3	4	3	2
3	4	6	4	3
4	6	8	6	4
3	4	6	4	3
2	3	4	3	2

The board above shows the number of possible ways to get to each of the 25 squares. There are only two ways to get to each of the corner squares. Therefore in order for the knight to visit each square once and only once he must either start on a corner square or finish on corner-square.

In this instance, the knight begins at the centre. Thus he must finish his journey on either of squares one, five, 21 or 25 (see below). To do this he must be careful not to visit squares eight, twelve, fourteen or eighteen (see below) until the later stages of his journey.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

### Suggestions for Teachers

**Point the way:** Suggest students start by determining for themselves all the possible ways to reach each of the 25 squares.

**Lead with questions:** How many possible ways for the knight to visit each of the outer corners? What will happen if the knight visits the corners early in his journey?

**Investigate further:** Is it possible for the knight to visit all the squares from a different starting point? Can all squares on the board be starting points?